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FORTRAN LIBRARY FOR THE SOLUTION OF ACOUSTIC PROBLEMS BY THE BOUNDARY ELEMENT METHOD

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Integrated Sound Software, [illegible]

[illegible]

Abstract

The boundary element method is a powerful tool in computational acoustic analysis. This paper is an introduction to the author's book **The Boundary Element Method in Acoustics** [12] and the accompanying software. Boundary element methods are outlined for three important classes of acoustic or Helmholtz problems: modelling the acoustic field either interior or exterior to a closed surface or carrying out an acoustic modal analysis. Each class of problem is considered in two, three dimensions and the axisymmetric case.

Fortran 77 codes implementing the methods are available. Test problems are used to demonstrate the methods. Examples of realistic application such as vehicle noise and loudspeaker design analysis illustrate the potential of the boundary element method in acoustic simulation.

INTRODUCTION

The boundary element method is a powerful technique in general acoustic simulation since only the boundaries or surfaces of radiating or scattering objects or enclosures need to be meshed. It is therefore often more efficient than the more typical finite element method. The on-line manual [14] provides a general introduction to the BEM.

Solutions to two-dimensional, three-dimensional and axisymmetric three-dimensional acoustic problems are developed in this work. In each case boundary element techniques are developed for the solution of interior and exterior boundary-value problems and the interior eigenvalue problem. A very general form of boundary condition is assumed and it is possible to include an incident field that is necessary in acoustic scattering, for example. Nine major Fortran subroutines are developed for solving each of these problems and the source codes are available, they are listed in table 1.

Table 1:The main subroutines

	Chapter	2-dimensional	3-dimensional	Axisymmetric
Core routines	3	H2LC	H3LC	H3ALC
Interior analysis	4	IBEM2	IBEM3	IBEMA
Exterior analysis	5	EBEM2	EBEM3	EBEMA
Modal analysis	6	MBEM2	MBEM3	MBEMA

The three subroutines, for each dimensional space, call a core routine that is responsible for discretising the integral equation. The core routines for 2D, 3D and axisymmetric problems require the surfaces to be divided into elements as illustrated in Figure 1. The methods employed in the core routines are described in Kirkup [12] or Kirkup [13].

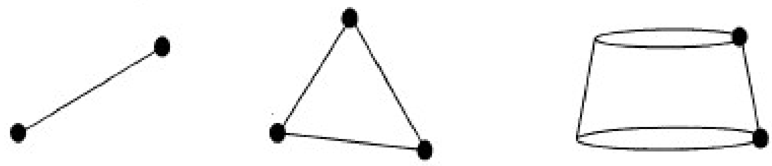

Figure 1. The straight line, planar triangle and truncated cone panels.

THE INTERIOR ACOUSTIC PROBLEM

The boundary element method (BEM) is a valuable technique in the solution of a range of physical and engineering problems (see www.boundary-element-method.co.uk). The BEM for the solution of the interior acoustic boundary-value problem have been developed in Meyer et al [15], Bernard et al [1], and Kipp and Bernard [6]. In this work the application of the BEM to the interior acoustic problem is developed further so that the solution with a general boundary, boundary condition and incident field can be obtained. The subroutine IBEM2 is demostrated by showing the results of applying it to a 2D car interior.

As an application to demonstrate the subroutine IBEM2, the boundary is a two-dimensional version of the vehicle interior used in Jeong-Guon [3]. A diagram of the vehicle interior is shown in Figure 2.

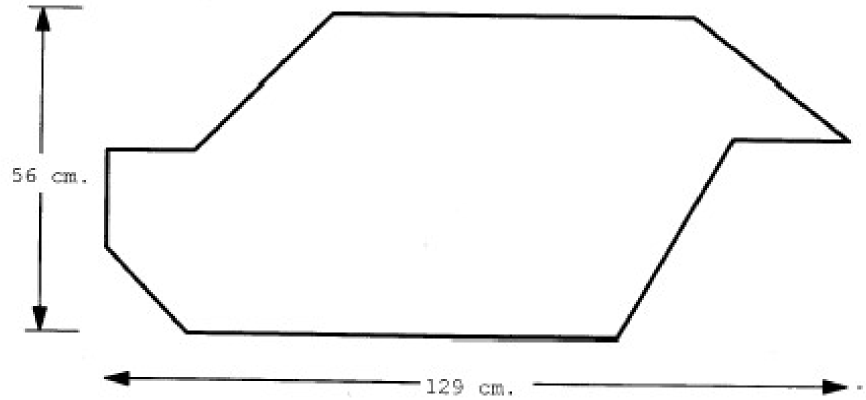

Figure 2. Diagram of the interior of the 2D car.

hence $k = 1.8265$. The numerical and exact solutions at the points (0,0,2), (0,0,4), (0,0,8) and (0,0,-2) are given in table 2.

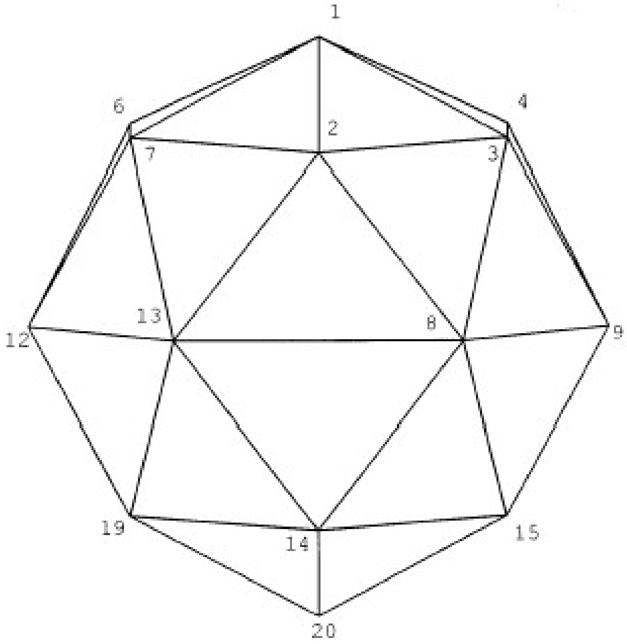

Figure 4. Representation of the sphere by flat triangular panels.

Table 2: Results from EBEM3_T

point	exact solution	numerical solution to Dirichlet condition	numerical solution to Neumann condition
(0,0,2)	-0.4360 - i 0.2447	-0.4628 - i 0.1897	-0.5011 - i 0.2389
(0,0,4)	0.1302 + i 0.2133	0.1557 + i 0.1960	0.1614 + i 0.2274
(0,0,8)	-0.0572 + i 0.1112	-0.0431 + i 0.1175	-0.0549+ i 0.1284
(0,0,-2)	-0.4360 - i 0.2447	-0.4628 - i 0.1897	-0.5011 - i 0.2389

Results from the application of this sort of method to the more practical problem of engine noise analysis are given in Kirkup and Tyrrell [9] and Kirkup [12].

INTERIOR MODAL ANALYSIS

An enclosed volume of fluid exhibits resonances in the same way as a structure does. At acoustic resonance frequencies excitation leads to a theoretically infinite response. In practice

The boundary is divided into 60 elements of approximately equal size. The test is run at a range of frequencies between 0 and 1000Hz. The boundary condition is defined so that the diagonal part of the boundary at the driver's feet is determined to have uniform vibration of amplitude 1mm at all frequencies. The remaining boundary is rigid. Figure 3 shows a graph of the computed sound pressure at the selected interior point (0.5,0.4) in the domain.

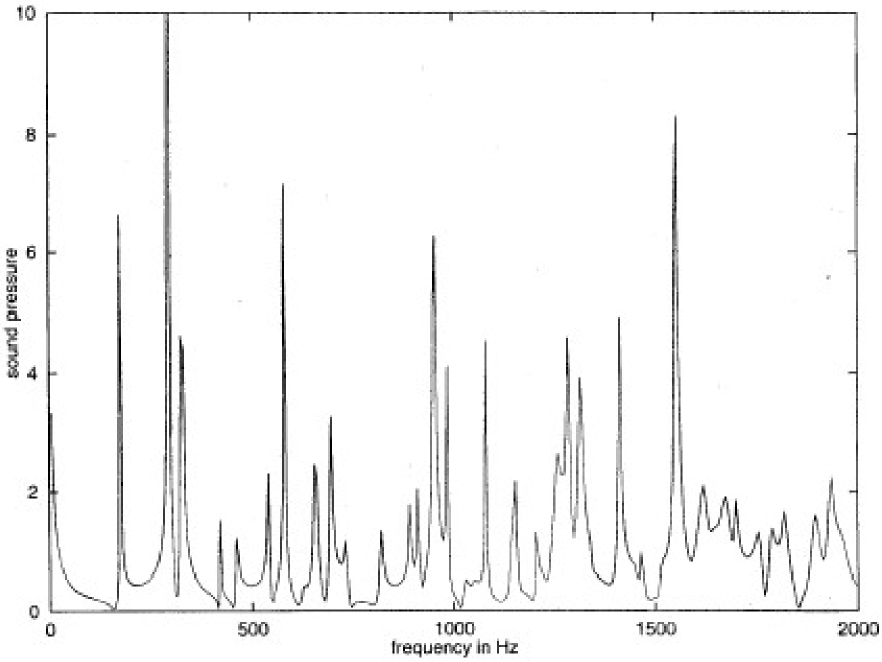

Figure 3. The magnitude of the sound pressure at (0.5,0.4).

THE EXTERIOR ACOUSTIC PROBLEM

Robust integral equation formulations of the exterior Helmholtz equation were developed in the 1960s and 1970s. The most important of these is the formulation of Burton and Miller [2]. The method is demonstated for example in Meyer et al [15]. It is this fomulation that is implemented in EBEM2, EBEM3 and EBEMA.

The test surface shown in figure 4 was developed in order to demonstrate subroutine EBEM3. It is a sphere that has been approximated by 36 planar triangles, as shown in figure 4. The velocity potential in the exterior is $\varphi = \frac{e^{ikr}}{r}$ which is a multiple of the Green's function and hence is clearly a solution of the Helmholtz equation. The acoustic frequency is 100Hz, the acoustic resonance frequencies inform us of the frequencies at which the acoustic response appears to be significantly magnified.

The resonant frequencies and the corresponding mode shapes are equivalent to the solutions of a non-linear algebraic eigenvalue problem in the BEM. By interpolating the matrices that arise in the boundary element method over a range of frequencies, the problem is reduced to that of solving a standard eigenvalue problem. This method was developed in Kirkup and Amini [10] and Kirkup and Jones [11] and also applied in Jeong-Guon [3].

In order to demonstrate the method a modal analysis of the air-tight interior of a test axially symmetric loudspeaker is carried out using subroutine MBEMA. Some of the results are compared with results from physical experiment. For further details on this application and the results obtained the reader is referred to [11], [12].

By applying the methods to an axisymmetric loudspeaker, the acoustic properties may be examined whilst reducing the dimension of the problem by one and thus reducing the computational expense when compared to the full three-dimensional analysis that is necessary for a general loudspeaker design. In addition, considerable previous work has been done on the structural vibration of axially symmetric loudspeaker drive units (see Jones [4] and Jones and Henwood [5], for example). In this work we show results for the cabinet shown in Figure 5, which is basically a cylinder 120mm deep and 132mm in diameter. The loudspeaker has a conical drive unit of radius 80mm fitted. Given these dimensions, the lowest cabinet resonance occurs around 1kHz, and the cabinet resonances can be observed in sound pressure measurements outside the cabinet.

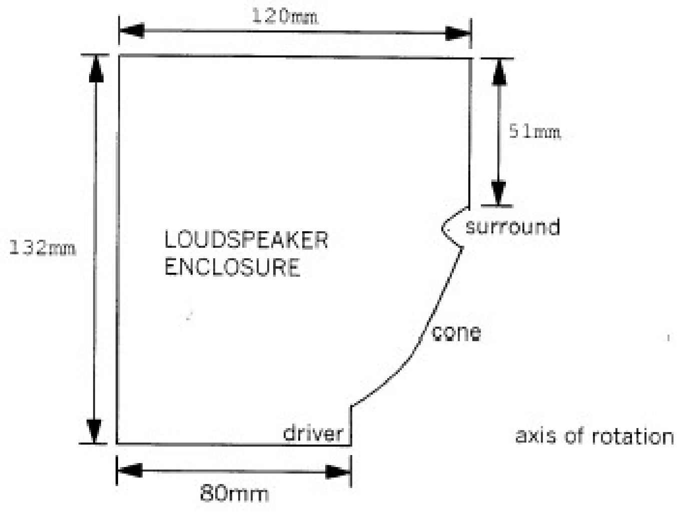

Fig. 5. Diagram of the axisymmetric cabinet.

For the application of the boundary element method the boundary of the generator of the loudspeaker is approximated by 32 conical elements of approximately equal length along the generator, as shown in Figure 5. Solutions were sought in the wavenumber ranges [0.0,5.0], [5.0,10.0], [10.0,15.0] and so on. In each range quadratic interpolation was applied. The approximations to the mode shapes are then obtained at around 100 points in the interior.

The five lowest resonant frequencies obtained through the boundary element methods and the results obtained by measurement are compared in table 3.

Table 3: Computed and measured loudspeaker resonant frequencies

Mode	Boundary Element	Experimental
1	1414 Hz	1318 Hz
2	1590 Hz	1679 Hz
3	2232 Hz	2133 Hz
4	2815 Hz	2691 Hz
5	2876 Hz	3306 Hz

The major contribution to the discrepancy between the measured and calculated values is believed to be the simplicity of the model chosen, which fails to include any internal structure to the loudspeaker. In addition, the maximum pressure occurs at slightly different frequencies for different microphone positions.

Figure 6 shows the third and fifth mode shapes obtained via the boundary element method. The mode shapes are constructed from the returned values in the domain. The values on the contours are arbitrary.

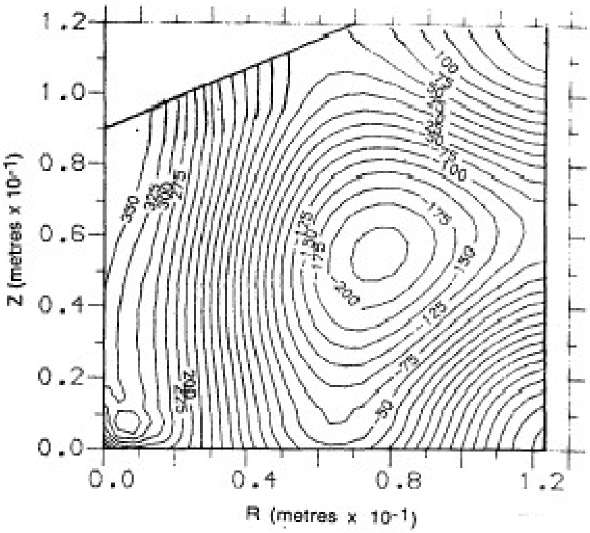

Fig 6(a). The third mode shape of the loudspeaker cabinet.

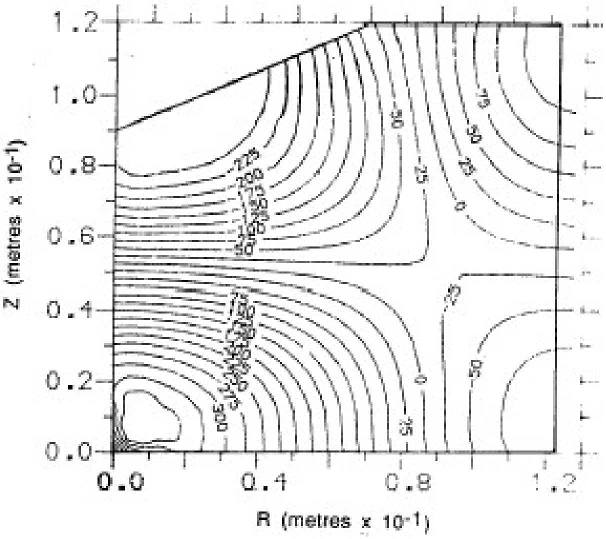

Fig 6(b). The fifth mode shape of the loudspeaker cabinet.

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